Risk, Return & CAPM

We will be expected to calculate

1. Expected Return (Average)
2. Variance
3. Standard Deviation

Variance = σ^2

- Population Data Formula: Probability-weighted average of squared deviations around the mean
- Sample Data Formula: Sum of squared deviations around the average return, divided by the number of observations minus 1

Standard Deviation = σ

- = $\sqrt{Variance}$

Example 1: Sample Data

Stock A had the following returns over the past five years. Calculate the following:

Year	Return
2010	10%
2011	12%
2012	14%
2013	16%
2014	18%

A) Calculate the expected return (average) of Stock A.
B) Calculate the variance and standard deviation of Stock A.

Risk, Return & CAPM

We will be expected to calculate

1. Expected Return (Average)
2. Variance
3. Standard Deviation

Example 2: Population Data

Assume that Stock B will have the following returns under each of the following three states of economy. Compute the expected return, variance, and standard deviation of the returns from Stock B.

State of Nature	Probability	Return
Boom	0.25	40%
Normal	0.50	20%
Recession	0.25	-10%

Computing the Weights of a Portfolio

- Portfolio weights refer to the percentage of the total investment that is made in each asset that is part of your portfolio
- The sum of the weights must equal 1
- The weights can be computed by dividing the portion of the total investment that is made in each asset
- A negative portfolio weight implies that we must short sell the asset for which the weight is negative

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Example 3: Portfolio Expected Return

Compute the expected return of the following portfolio of two stocks.

Investment	Dollar Invested	Return
Stock A	$10,000	5%
Stock B	$15,000	10%

Risk, Return & CAPM

Example 4: Portfolio Expected Return

XYZ Inc. common stock has a return of 20%, while the common stock of ABC Company has an expected return of 10%.

You want to create a portfolio that has an expected return of 14.4% by investing $4,000 in ABC Co. some amount in XYZ stock and the remainder in a risk-free asset which has a return of 4%. If you have $10,000 in total to purchase the securities how much will she invest in XYZ common stock?

Example 5: Portfolio Standard Deviation

Stocks P and Q have the following returns in three different scenarios. You build a portfolio by investing 60% of your money in stock P and the remaining in stock Q. Compute the standard deviation (risk) of the following portfolio of two stocks?

State of Nature	Probability	Return Stock P	Return Stock Q	
Boom	0.2	12%	5%	
Normal	0.6	7%	3%	
Recession	0.2	3%	-1%	

Risk, Return & CAPM

Portfolio of Stocks: How do we choose the stocks in a portfolio?

- In simple terms, when we build a portfolio of stocks it would be useful to know the relationship between two stocks.
- This will help us make better decisions with respect to risk. We will discuss this later.
- We have **two** measures to help identify and understand how two stocks perform with respect to each other
  - Covariance
  - Correlation Coefficient

Covariance

- The covariance of two stocks is a concept that measures how much the returns of two stocks move together.
- A negative covariance means the stocks move differently, while a positive means they move similarly
- Difficult to interpret and compare, as there is no distinct range. You can get results that are high or low positive values or results that are high or low negative values.

Covariance Formulas

Covariance: $\text{Cov}(r_j, r_m) = \rho_{j,m}\sigma_j\sigma_m$

Covariance: the probability-weighted average of cross-products of the deviations from the respective means

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Example 6 : Covariance

Stocks P and Q have the following returns in three different scenarios. You build a portfolio by investing 35% of your money in stock P and the remaining in stock Q.

What is the covariance between stock P and Q?

Scenario	Probability	Return P	Return Q
Boom	0.35	16%	1%
Normal	0.45	6%	7%
Recession	0.2	-4%	1%

The Correlation Coefficient (ρ – Rho)

- Measures the strength and the direction of the relationship between two variables
- Correlation coefficient is a standardized measure, must be between **-1 and 1.**
- If positive (0 to 1) they move together. The closer to +1 the stronger the correlation.
- If negative (-1 to 0) they do not move together. The closer to -1, the less correlated the relationship.

The Correlation Coefficient Formulas

Correlation: $\rho_{j,m} = \frac{\text{Cov}(r_j, r_m)}{\sigma_j \sigma_m}$

Example 7 : Correlation Coefficient

Given the same information from example 6, calculate the correlation coefficient between stock P and Q?

Scenario	Probability	Return P	Return Q
Boom	0.35	16%	1%
Normal	0.45	6%	7%
Recession	0.2	-4%	1%

Additional information provided:
Std Dev(Stock P) = 3.0%
Std Dev(Stock Q) = 7.3%

Risk, Return & CAPM

Example 8 : Portfolio Standard Deviation - Another Look

Assume the returns of two stocks (A & B) move in the exact opposite direction. Based on the information provided below, what should be the std deviation of the portfolio return?

Stock	Std. Dev.	Portfolio Weight
A	13%	0.41
B	22%	0.59

$$\sigma_P = \sqrt{x_1^2\sigma_1^2 + x_2^2\sigma_2^2 + 2x_1x_2\rho_{1,2}\sigma_1\sigma_2}$$

x_1* and *x_2 are fractions of the portfolio in the first and second assets, respectively

σ_1 and σ_2 are standard deviations of the returns on the first and second assets, respectively

$\rho_{1,2}$ is the correlation coefficient for the first and second assets

Risk, Return & CAPM

Example 9: Market Risk Premium

The risk free rate is 5% and the average return on the market is 10% then what is the Market Risk Premium?

$$\text{Beta of stock } j: \qquad \beta_j = \frac{\rho_{j,m}\sigma_j}{\sigma_m} = \frac{\text{Cov}(r_j, r_m)}{\sigma_m^2}$$

Example 10: Beta of a Stock

XYZ Inc. common stock and its correlation coefficient with the market is 0.748 . The standard deviation of the stock 9.58%, and the standard deviation of the markets return is 3.25%. Calculate the beta of XYZ stock.

Example 11: Beta of a Portfolio

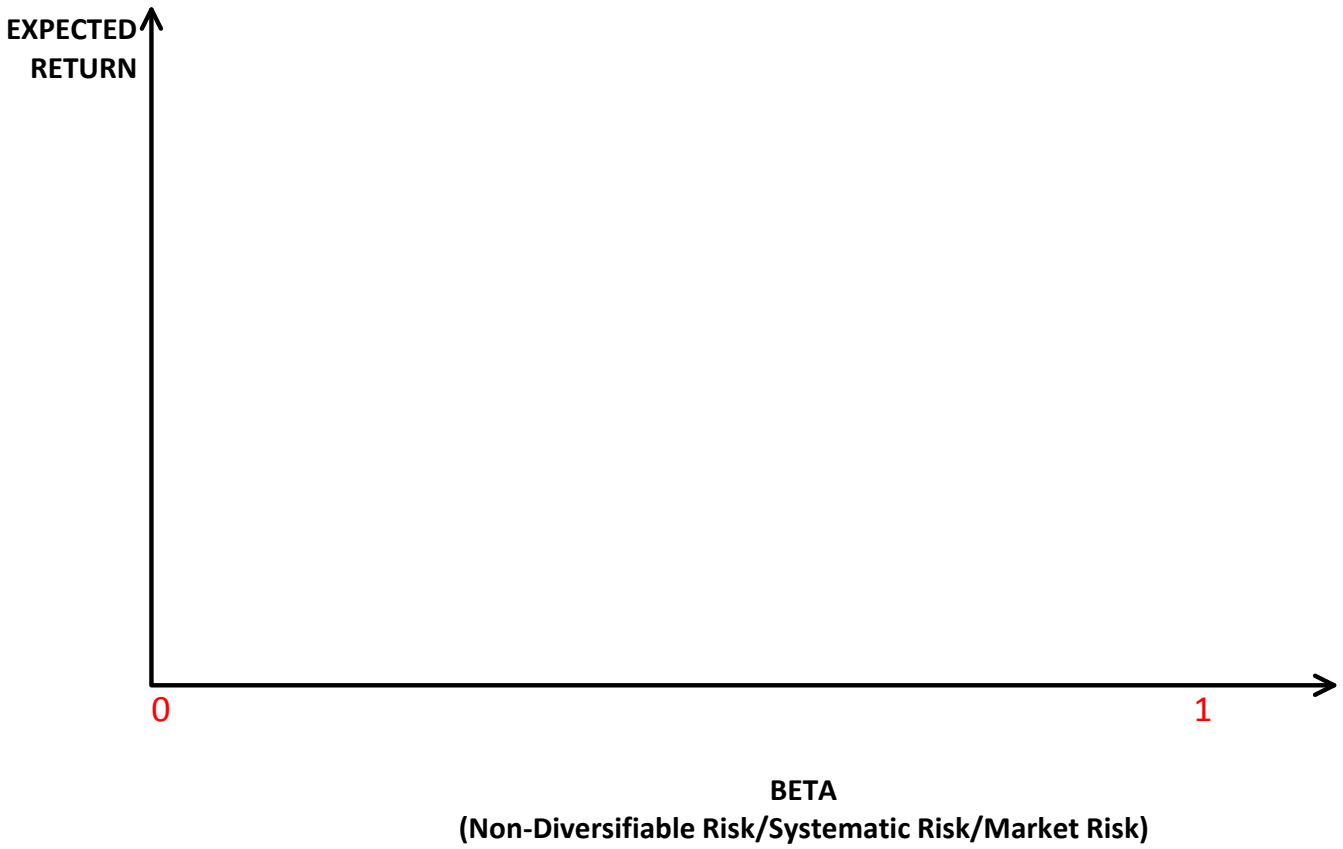
Assume that you invested 20% of your wealth in Stock A which is twice as risky as the market, 10% of your wealth in the market portfolio, 25% of your wealth in a risk free asset and the remainder in Stock B which has a beta of 0.8. What is the beta of your portfolio?

Risk, Return & CAPM

CAPM - Calculating the Required Return

- We know the Return Required on an investment is determined as a function of the risk level of that investment
- We also know investors are compensated only for the risk that cannot be diversified (Non-Diversifiable Risk)
- This risk is measure by Beta
- How can this new concept of risk, Beta, help us calculate the expected return that an investment should require

Security Market Line (SML)



Risk, Return & CAPM

The capital asset pricing model (CAPM):

Expected return = Risk-free rate + Asset risk premium ➔ $r = r_f + \beta(r_m - r_f)$.

Example 12: CAPM & Required Return

If the beta for XYZ Inc. is 1.2 and average return on the market is 9%. What should be the required return of XYZ Inc. stock if the Risk free rate is 4%?

Risk, Return & CAPM

The capital asset pricing model (CAPM):

Expected return = Risk-free rate + Asset risk premium ➔ $r = r_f + \beta(r_m - r_f)$.

Example 13: CAPM & Required Return

What should be the required return if XYZ Inc. stock is just as risky as the market, the risk free rate is 3%, and the MRP is 5%?

Example 14: Correctly/Incorrectly Priced

Which of the following stocks is (are) incorrectly priced if the risk-free rate is 4% and the market risk premium is 6%?

Stock	A	B	C
Beta	1.25	0.80	1.06
Expected Return	12.60%	8.80%	11.20%

Example 15: Beta of a Stock

XYZ Inc. common stock has an expected return of 15% and the risk free rate is 5%. If the average return on the market is 10% then what is the beta of XYZ Inc. common stock?

Example 16:

When the overall market experiences a decline of 8%, an investor with a portfolio that has a beta of 1.5, will most likely incur a return that is:

A) negative, but less than 8%.
B) negative, and greater than 8%.
C) positive, but less than 8%.
D) positive, and more than 8%.
E) exactly 8%.

Risk, Return & CAPM

Example 17:

An investor was expecting an 18 percent return on her portfolio with beta of 1.25 before the market risk premium increased from 8 percent to 10 percent. Based on this change, what return will now be expected on the portfolio?